Appendix D The Decibel

Telephone engineers who were concerned with the power loss across the cascaded circuits used to transmit telephone signals introduced the decibel. Figure D.1 defines the problem.

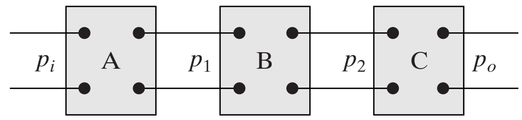

Figure D.1 ▲ Three cascaded circuits.

There, p_i is the power input to the system, p_1 is the power output of circuit A, p_2 is the power output of circuit B, and p_o is the power output of the system. The power gain of each circuit is the ratio of the power out to the power in. Thus

$$\sigma_A = \frac{p_1}{p_i}, \quad \sigma_B = \frac{p_2}{p_1}, \quad \text{and} \quad \sigma_C = \frac{p_o}{p_2}.$$

The overall power gain of the system is simply the product of the individual gains, or

$$\frac{p_o}{p_i} = \frac{p_1}{p_i}\frac{p_2}{p_1}\frac{p_o}{p_2} = \sigma_A\sigma_B\sigma_C.$$

The multiplication of power ratios is converted to addition by means of the logarithm; that is,

$$\log_{10}\frac{p_o}{p_i} = \log_{10}\sigma_A + \log_{10}\sigma_B + \log_{10}\sigma_C.$$

This log ratio of the powers was named the **bel**, in honor of Alexander Graham Bell. Thus we calculate the overall power gain, in bels, simply by summing the power gains, also in bels, of each segment of the transmission system. In practice, the bel is an inconveniently large quantity. One-tenth of a bel is a more useful measure of power gain; hence the **decibel**. The number of decibels equals 10 times the number of bels, so

$$\text{Number of decibels} = 10\log_{10}\frac{p_o}{p_i}.$$

When we use the decibel as a measure of power ratios, in some situations the resistance seen looking into the circuit equals the resistance loading the circuit, as illustrated in Fig. D.2.

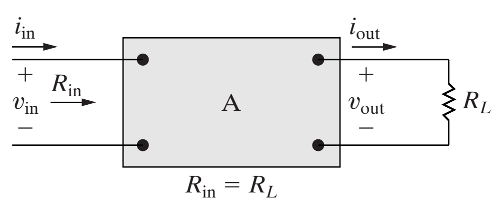

Figure D.2 ▲ A circuit in which the input resistance equals the load resistance.

When the input resistance equals the load resistance, we can convert the power ratio to either a voltage ratio or a current ratio:

$$\frac{p_o}{p_i} = \frac{v_{out}^2/R_L}{v_{in}^2/R_{in}} = \left(\frac{v_{out}}{v_{in}}\right)^2$$

or

$$\frac{p_o}{p_i} = \frac{i_{out}^2 R_L}{i_{in}^2 R_{in}} = \left(\frac{i_{out}}{i_{in}}\right)^2.$$

These equations show that the number of decibels becomes

$$\text{Number of decibels} = 20 \log_{10} \frac{v_{out}}{v_{in}}$$

$$= 20 \log_{10} \frac{i_{out}}{i_{in}}. \qquad \text{(D.1)}$$

The definition of the decibel used in Bode diagrams (see Appendix E) is borrowed from the results expressed by Eq. D.1, since these results apply to any transfer function involving a voltage ratio, a current ratio, a voltage-to-current ratio, or a current-to-voltage ratio. You should keep the original definition of the decibel firmly in mind because it is of fundamental importance in many engineering applications.

When you are working with transfer function amplitudes expressed in decibels, having a table that translates the decibel value to the actual value of the output/input ratio is helpful. Table D.1 gives some useful pairs. The ratio corresponding to a negative decibel value is the reciprocal of the positive ratio. For example, −3 dB corresponds to an output/input ratio of 1/1.41, or 0.707. Interestingly, −3 dB corresponds to the half-power frequencies of the filter circuits discussed in Chapters 14 and 15.

TABLE D.1 Some dB-Ratio Pairs

dB	Ratio	dB	Ratio
0	1.00	30	31.62
3	1.41	40	100.00
6	2.00	60	10^3
10	3.16	80	10^4
15	5.62	100	10^5
20	10.00	120	10^6

The decibel is also used as a unit of power when it expresses the ratio of a known power to a reference power. Usually the reference power is 1 mW and the power unit is written dBm, which stands for "decibels relative to one milliwatt." For example, a power of 20 mW corresponds to ±13 dBm.

AC voltmeters commonly provide dBm readings that assume not only a 1 mW reference power but also a 600 Ω reference resistance (a value commonly used in telephone systems). Since a power of 1 mW in 600 Ω corresponds to 0.7746 V (rms), that voltage is read as 0 dBm on the meter. For analog meters, there usually is exactly a 10 dB difference between adjacent ranges. Although the scales may be marked 0.1, 0.3, 1, 3, 10, and so on, in fact 3.16 V on the 3 V scale lines up with 1 V on the 1 V scale.

Some voltmeters provide a switch to choose a reference resistance (50, 135, 600, or 900 Ω) or to select dBm or dBV (decibels relative to one volt).