Microelectronics Equations

I. Introduction

This document contains various equations and formulas that are related to electrical circuits. It is written using Overleaf which is an online LaTeX editor. If you see any error please email jacobwatts@mail.weber.edu. This document contains many equations for finding different values within electrical circuits.

II. Physical Constants

Constant	Symbol	Rounded Value
elementary charge	e	$1.6022 \times 10^{-19}\ C$
speed of light in vacuum	c	$2.9979 \times 10^{8}\ m/s$
electric constant	ϵ_0	$8.8542 \times 10^{-12}\ F/m$
magnetic constant	μ_0	$4\pi \times 10^{-7}\ H/m$
Planck constant	h	$6.626 \times 10^{-34}\ J \cdot s$
Boltzmann constant	k	$1.381 \times 10^{-23}\ J/K$
Farady constant	F	$9.649 \times 10^{4}\ C/mol$
proton gyromagnetic ratio	γ_p	2.6752×10^{8}
standard acceleration of free fall	g_n	$9.80665\ m/s^2$
standard atmosphere	atm	$1013.25\ Pa$

III. Engineering Prefixes

It is important to know how to convert between different scientific prefixes. For example when given 12×10^3 A and you would like to know what the unit is. Use the following table.

A. SI Units Chart

Prefix	Symbol	Exponential	Multiplier
$tera$	T	10^{12}	$1,000,000,000,000$
$giga$	G	10^{9}	$1,000,000,000$
$mega$	M	10^{6}	$1,000,000$
$kilo$	k	10^{3}	$1,000$
$None$	–	10^{0}	1
$centi$	c	10^{-2}	0.01
$milli$	m	10^{-3}	0.001
$micro$	μ	10^{-6}	0.000001
$nano$	n	10^{-9}	0.000000001
$pico$	p	10^{-12}	0.000000000001

IV. Ohm's Law

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points.

The following equations are the relationships between V, R, and I. Looking at the triangle it is easily recognizable the

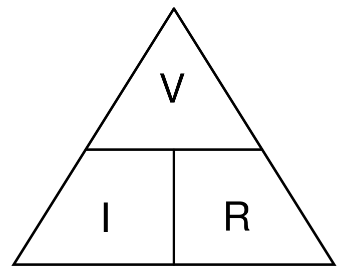


Fig. 1: Ohm's Law Triangle

for the value that you are trying to find you set that equal to whatever orientation the other two values are in.

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I} \tag{1}$$

V. Resistors

A. Resistors in Series

To calculate the resistance of multiple resistors in series, simply take the sum of all the resistors. Please refer to Equation 11.

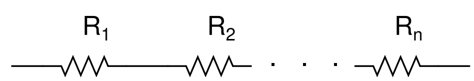


Fig. 2: Resistors in Series

$$R_T = R_1 + R_2 + ...\ R_n \tag{2}$$

B. Two (2) Resistors in Parallel

To calculate the resistance of two (2) resistors in series. Please refer to Equation 3.

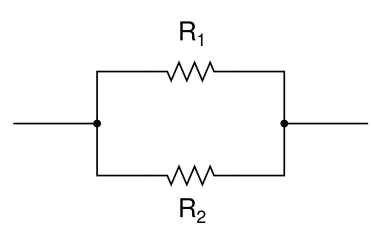


Fig. 3: Resistors in Parallel

$$R = \frac{R_1 R_2}{R_1 + R_2} \tag{3}$$

C. Multiple Resistors in Parallel

Please see Figure 4. You will notice that it has more than two (2) resistors. If there are more than two (2) resistors you will be unable to use Equation 3. So, the Equation 4 is used to calculate the total resistance. An important note to remember is that Equation 4 can still be used even when there are only two (2) resistors. I need to add more information here because I want the equation to be under the graphic like so, but that still is not enough information so I will continue to type until it is there. I don't understand why the equation won't post below this sentence.

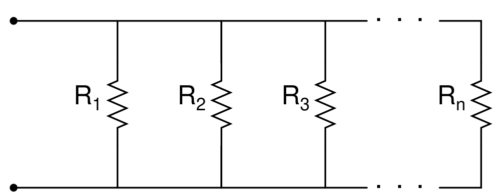
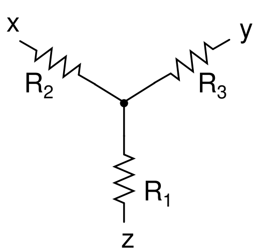


Fig. 4: Multiple resistors in parallel

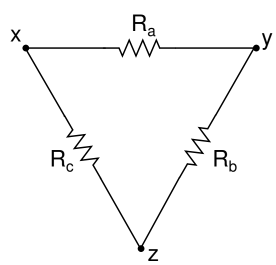
$$R_P = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots \frac{1}{R_n}} \tag{4}$$

D. Wye to Delta Transformation

The following figures and equations will illustrate how to convert from Wye to Delta and vice versa.



(a) A subfigure



(b) A subfigure

Fig. 5: A figure with two subfigures

$$\begin{aligned} R_1 &= \frac{R_c\, R_b}{R_a + R_b + R_c} \\ R_2 &= \frac{R_a\, R_c}{R_a + R_b + R_c} \\ R_3 &= \frac{R_a\, R_b}{R_a + R_b + R_c} \end{aligned} \tag{5}$$

$$\begin{aligned} R_a &= \frac{R_1\, R_2 + R_2\, R_3 + R_3\, R_1}{R_1} \\ R_b &= \frac{R_1\, R_2 + R_2\, R_3 + R_3\, R_1}{R_2} \\ R_c &= \frac{R_1\, R_2 + R_2\, R_3 + R_3\, R_1}{R_3} \end{aligned} \tag{6}$$

VI. Current and Voltage Dividers

It is important to understand how to find the voltage and current in circuits. This because you need to figure out how to put the current into the circuit. I don't understand why the image isn't moving down. I still don't know why it's not moving down still. I wonder if I need to look it up online.

A. Current Divider

An example of how to use the current divider equation.

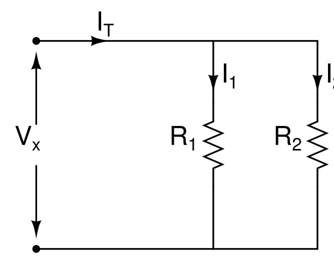


Fig. 6: Current Divider circuit

To find the current through on of the branches. First(1) find the R_p and plug it into equation (5) to find the current through the branch.

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \qquad I_1 = \frac{R_P}{R_1}(I_T) \tag{7}$$

The following equation is also a current divider equation but can only be used for circuits with two (2) resistors.

$$I_1 = \frac{I_T R_2}{R_1 + R_2} \tag{8}$$

B. Voltage Divider

An example of how to use the voltage divider equation

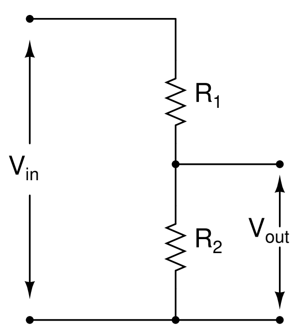


Fig. 7: Voltage Divider Circuit

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in} \tag{9}$$

VII. Capacitors

A. Capacitors in Series

Capacitors in Series. Basically just do the same as resistors in parallel but obviously use the values found for the capacitors.

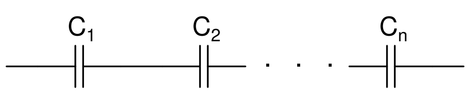


Fig. 8: Caption

$$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ... \frac{1}{C_n}} \tag{10}$$

B. Capacitors in Parallel

Capacitors in Parallel

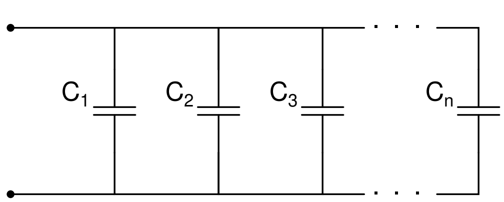


Fig. 9: Caption

$$C_P = C_1 + C_2 + ... \; C_n \tag{11}$$

VIII. Operation Amplifiers (OP)

A. Inverting Op-amp

To find the voltage gain of an Inverting Op-amp use the following equation:

$$A_v = \frac{-R_F}{R_{in}} \tag{12}$$

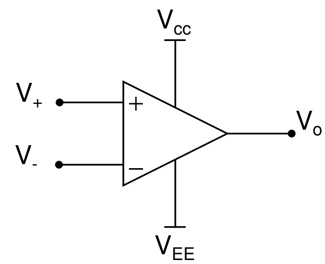


Fig. 10: Operation Amplifier

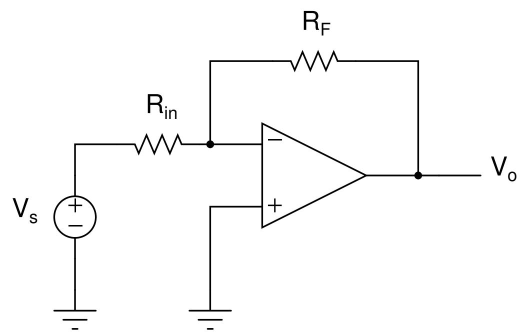


Fig. 11: Inverting Op amp

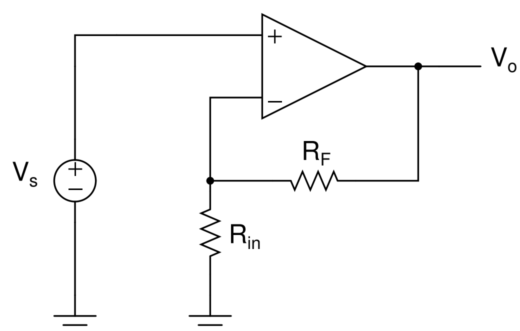


Fig. 12: Non-inverting Op amp

B. Non Inverting Op-amp

To find the voltage gain of a Non-inverting Op-amp use the following equation:

$$A_v = 1 + \frac{R_F}{R_i n} \tag{13}$$

C. Ideal Op-Amp Voltage gain

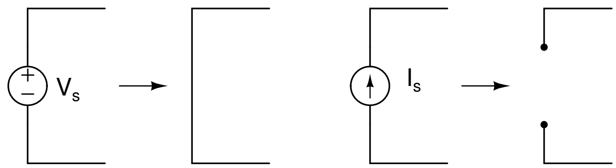
The ideal Op-Amp will have the following characteristics

$$\begin{aligned} A_o &\to \infty \\ R_{in} &\to \infty \\ R_o &\to 0 \\ i_+ = i_- &= 0 \\ V_- &= V_+ \end{aligned} \tag{14}$$

IX. Additional Analysis Techniques

A. Superposition

To calculate superposition short circuit the voltage source, and calculate $V_0^{'}$. Then replace the voltage source and break the current source, then calculate $V_0^{''}$.



(a) Voltage Source (b) Current Source

Fig. 13: Superposition Calculations

B. Thévenin's Equivalent

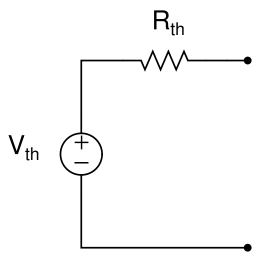


Fig. 14: Thevenin's Equivalent

C. Norton's Equivalent

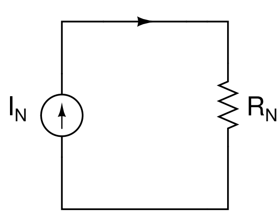


Fig. 15: Norton's Equivalent

X. Capacitance and Inductance

A. Capacitance

A capacitor is built out of parallel plates and a dielectric substance. Charge storage in a capacitor is found by using the following equation.

$$q = CV \tag{15}$$

B. Inductors

Inductors store energy.

C. Inductors in Series

$$L_{eq} = L_1 + L_2 + ... \ L_n \tag{16}$$

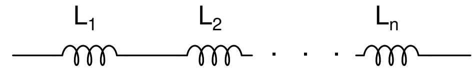


Fig. 16: Inductors in Series

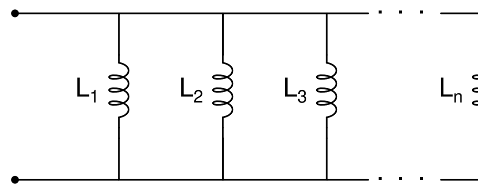


Fig. 17: Inductors in Parallel

D. Inductors in Parallel

$$L_P = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} ... \frac{1}{L_n}} \tag{17}$$

XI. Digital Circuits

A. Digital-to-Analog Conversion

The behavior of the DAC can be expressed mathematically as:

$$\begin{aligned} v_0 = (b_1 2^{-1} + b_2 2^{-2} + ... + b_n 2^{-n})V_{FS} \\ for \ \ b_i \in \{1, 0\} \end{aligned} \tag{18}$$

Example: A 10-bit D/A converter has $V_{FS} = 5.12V$. What is the output voltage for a binary input code of (1100010001)? What is V_{LSB}? What is the size of the MSB?

Answers: 3.925 V; 5 mV; 2.56 V

1) Quantization Error:

$$v_\epsilon = |v_X - (b_1 2^{-1} = b_2 2^{-2} + ... + b_n 2^{-n})V_{FS}| \tag{19}$$

XII. Problem-Solving Approach

1) State the problem as clearly as possible.
2) List the known information and given data.
3) Define the unknowns that must be found to solve the problem.
4) Develop an approach from a group of possible alternatives.
5) Perform an analysis to find a solution to the problem. As part of the analysis, be sure to draw the circuit and label the variables.
6) Check the results. Has the problem been solved? Is the math correct? Have all the unknowns been found? Have the assumptions been satisfied? Do the results satisfy simple consistency checks?
7) Evaluate the solution. Is the solution realistic? Can it be built? If not, repeat steps 4-7 until a satisfactory solution is obtained.
8) Computer-aided analysis. SPICE and other computer tools are highly useful to check the results and to see if

the solution satisfies the problem requirements. Compare the computer results to your hand results.

XIII. Solid-State Electronics

A. Electrical Classification of Solid Materials

MATERIALS	RESISTIVITY
Insulators	$10^5 < \rho$
Semiconductors	$10^{-3} < \rho < 10^5$
Conductors	$\rho < 10^{-3}$

B. Semiconductor Materials

SEMICONDUCTOR	BANDGAP ENERGY $E_G(eV)$
Carbon (diamond)	5.47
Silicon	1.12
Germanium	0.66
Tin	0.082
Gallium arsenide	1.42
Gallium nitride	3.49
Indium phosphide	1.35
Boron nitride	7.50
Silicon carbide	3.26
Silicon germanium	1.10
Cadmium selenide	1.70

C. Intrinsic Carrier Density

$$n_i^2 = BT^3\, exp(-\frac{E_G}{kT}) \tag{20}$$

where E_G = semiconductor bandgap energy in eV (electron volts)
k = Boltzmann's constant, 8.62×10^{-5} eV/K
T = absolute temperature, K
B = material-dependent parameter, $1.08 \times 10^{31}\ forSi$

For simplicity, in subsequent calculations we use

$$n_i = 10^{10}/cm^3 \tag{21}$$

as the room temperature value of n_i for silicon.

D. Mobility

We know from electromagnetics that charged particles move in response to an applied electric field. This movement is termed drift, and the resulting current flow is know as drift current.

$$\mathbf{v}_n = -\mu_n \mathbf{E} \tag{22}$$

$$\mathbf{v}_p = -\mu_p \mathbf{E} \tag{23}$$

where
$\mathbf{v}_n$ = velocity of electrons (cm/s)
$\mathbf{v}_p$ = velocity of holes (cm/s)
μ_n = **electron mobility**, 1420 $cm^2/V\cdot$ s in intrinsic Si
μ_p = **hole mobility**, 470 $cm^2/V\cdot$ s in intrinsic Si

E. Resistivity of Intrinsic Silicon

$$\begin{aligned} j_n^{drift} &= Q_n v_n = (-qn)(-\mu_n E) = qn\mu_n E \quad A/cm^2 \\ j_p^{drift} &= Q_p v_p = (-qp)(-\mu_p E) = qp\mu_p E \quad A/cm^2 \end{aligned} \tag{24}$$

The total drift current density is then given by:

$$j_T^{drift} = j_n + j_p = q(n\mu_n + p\mu_p)E = \sigma E \tag{25}$$

The equation defines σ, the **electrical conductivity:**

$$\sigma = q(n\mu_n + p\mu_p) \qquad (\Omega \cdot cm)^{-1} \tag{26}$$

Resistivity ρ is the reciprocal of conductivity:

$$\rho = \frac{1}{\sigma} \qquad (\Omega \cdot cm) \tag{27}$$

$$R = \frac{\rho L}{A}\Omega \tag{28}$$

where
ρ = resistivity of material
L = length of material
A = area of material

F. Impurities in Semiconductors

$$\begin{aligned} N_D &= \textbf{donor impurity concentration} \qquad \text{atoms}/cm^3 \\ n &= \text{electron concentration} \qquad /cm^3 \end{aligned} \tag{29}$$

1) Donor Impurities in Silicon: **Donor impurities** in silicon are from column V, having five valence electrons in the outer shell. The most commonly used elements are phosphorus, arsenic, and antimony.

$$\begin{aligned} N_A &= \textbf{acceptor impurity concentration} \qquad \text{atoms}/cm^3 \\ p &= \text{hole concentration} \qquad /cm^3 \end{aligned} \tag{30}$$

2) Acceptor Impurities in Silicon: **Acceptor impurities** in silicon are from column III and have one less electron than silicon in the outer shell. The primary acceptor impurity is boron.

3) Practical Doping Levels:

$$\begin{aligned} &\text{For } \textbf{n-type } (N_D > N_A): \quad n \cong N_D - N_A \quad p = \frac{n_i^2}{N_D - N_A} \\ &\text{For } \textbf{p-type } (N_A > N_D): \quad p \cong N_A - N_D \quad n = \frac{n_i^2}{N_A - N_D} \end{aligned} \tag{31}$$

Typical values of doping fall in this range:

$$10^{14}\ /cm^3 \leqslant |N_A - N_D| \leqslant 10^{21}\ /cm^3 \tag{32}$$

4) *Mobility Equations:*

$$\mu_n = 52.2 + \frac{1365}{1 + (\frac{N_T}{9.68 \times 10^{16}})^{0.68}}$$
$$\mu_p = 44.9 + \frac{426}{1 + (\frac{N_T}{2.23 \times 10^{16}})^{0.72}} \tag{33}$$

5) *Thermal Voltage V_T:*

$$V_T = kT/q = 0.0258V \text{ at } 300K \tag{34}$$

XIV. Solid-State Diodes and Diode Circuits

A. *The pn Junction Diode*

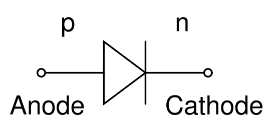


Fig. 18: Diode circuit symbol

Junction Potential ϕ_j

$$\phi_j = V_T ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{35}$$