

# Electrical Circuits Equations

## I. INTRODUCTION

This document contains various equations and formulas that are related to electrical circuits. It is written using Overleaf which is an online  $\text{\LaTeX}$  editor. If you see any error please email [jacobwatts@mail.weber.edu](mailto:jacobwatts@mail.weber.edu). This document contains many equations for finding different values within electrical circuits.

## II. ENGINEERING PREFIXES

It is important to know how to convert between different scientific prefixes. For example when given  $12 \times 10^3$  A and you would like to know what the unit is. Use the following table.

### A. SI Units Chart

Prefix	Symbol	Exponential	Multiplier
<i>tera</i>	T	$10^{12}$	1,000,000,000,000
<i>giga</i>	G	$10^9$	1,000,000,000
<i>mega</i>	M	$10^6$	1,000,000
<i>kilo</i>	k	$10^3$	1,000
<i>None</i>	-	$10^0$	1
<i>milli</i>	m	$10^{-3}$	0.001
<i>micro</i>	$\mu$	$10^{-6}$	0.000001
<i>nano</i>	n	$10^{-9}$	0.000000001
<i>pico</i>	p	$10^{-12}$	0.000000000001

## III. OHM'S LAW

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points.

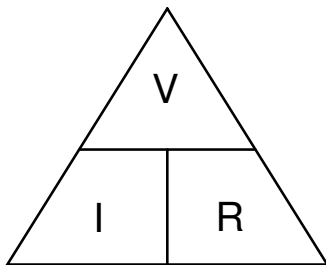


Fig. 1: Ohm's Law Triangle

The following equations are the relationships between  $V$ ,  $R$ , and  $I$ . Looking at the triangle it is easily recognizable the for the value that you are trying to find you set that equal to whatever orientation the other two values are in.

$$V = IR \quad I = \frac{V}{R} \quad R = \frac{V}{I} \quad (1)$$

## IV. RESISTORS

### A. Resistors in Series

To calculate the resistance of multiple resistors in series, simply take the sum of all the resistors. Please refer to Equation 11.

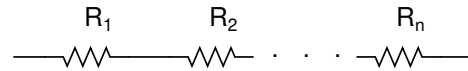


Fig. 2: Resistors in Series

$$R_T = R_1 + R_2 + \dots R_n \quad (2)$$

### B. Two (2) Resistors in Parallel

To calculate the resistance of two (2) resistors in series. Please refer to Equation 3.

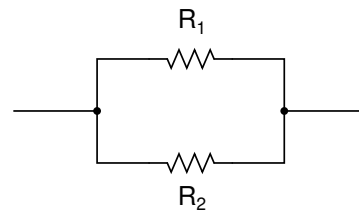


Fig. 3: Resistors in Parallel

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad (3)$$

### C. Multiple Resistors in Parallel

Please see Figure 4. You will notice that it has more than two (2) resistors. If there are more than two (2) resistors you will be unable to use Equation 3.

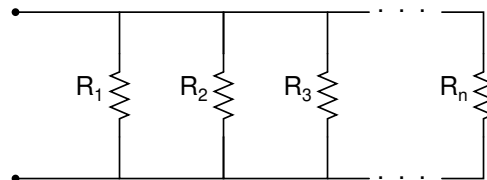


Fig. 4: Multiple resistors in parallel

$$R_P = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots \frac{1}{R_n}} \quad (4)$$

## V. CURRENT AND VOLTAGE DIVIDERS

It is important to understand how to find the voltage and current in circuits. This because you need to figure out how to put the current into the circuit. I don't understand why the image isn't moving down. I still don't know why it's not moving down still. I wonder if I need to look it up online.

### A. Current Divider

An example of how to use the current divider equation. To

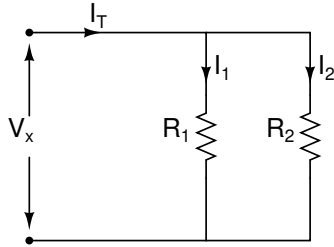


Fig. 5: Current Divider circuit

find the current through one of the branches. First(1) find the  $R_p$  and plug it into equation (5) to find the current through the branch.

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \quad I_1 = \frac{R_p}{R_1} (I_T) \quad (5)$$

The following equation is also a current divider equation but can only be used for circuits with two (2) resistors.

$$I_1 = \frac{I_T R_2}{R_1 + R_2} \quad (6)$$

### B. Voltage Divider

An example of how to use the voltage divider equation

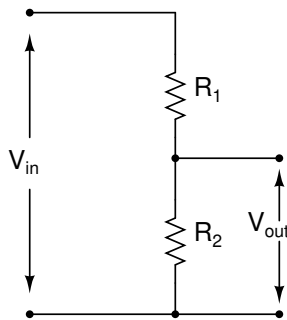
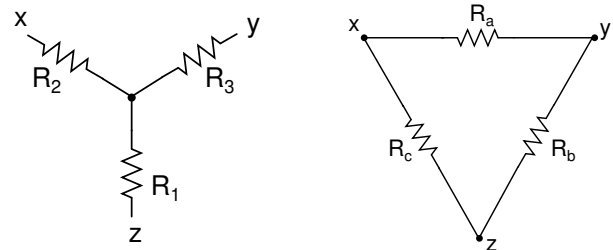


Fig. 6: Voltage Divider Circuit

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in} \quad (7)$$

### C. Wye to Delta Transformation

The following figures and equations will illustrate how to convert from Wye to Delta and vice versa.



(a) Wye Network

(b) Delta Network

Fig. 7: Wye and Delta resistor network

$$R_1 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \quad (8)$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \quad (9)$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

## VI. CAPACITORS

### A. Capacitors in Series

Capacitors in Series. Basically just do the same as resistors in parallel but obviously use the values found for the capacitors.

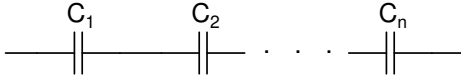


Fig. 8: Capacitors in Series

$$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots \frac{1}{C_n}} \quad (10)$$

### B. Capacitors in Parallel

Capacitors in parallel are treated just like resistors in series. Just sum up the capacitor values.

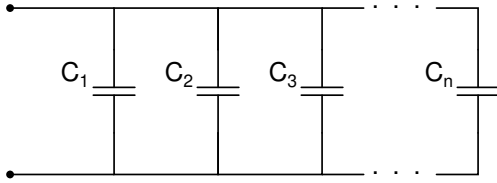


Fig. 9: Capacitors in Parallel

$$C_P = C_1 + C_2 + C_3 \dots C_n \quad (11)$$

### C. Terminal Equations for Ideal Capacitors

Capacitance is a linear circuit parameter that relates the current induced by a time-varying electric field to the voltage producing the field. Capacitors are passive elements; they can store and release energy, but they cannot generate or dissipate energy. A capacitor:

- does not permit an instantaneous change in its terminal **voltage**,
- does permit an instantaneous change in its terminal **current**, and
- behaves as an open circuit in the presence of a constant terminal voltage.

$$v = \frac{1}{C} \int_0^t i dt + v(0) \quad (\text{V}) \quad (12)$$

$$i = C \frac{dv}{dt} \quad (\text{A}) \quad (13)$$

$$p = vi = Cv \frac{dv}{dt} \quad (\text{W}) \quad (14)$$

$$w = \frac{1}{2} C v^2 \quad (\text{J}) \quad (15)$$

## VII. INDUCTORS

### A. Inductors

Inductors store energy in the form of a magnetic field.

### B. Inductors in Series

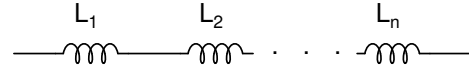


Fig. 10: Inductors in Series

$$L_{eq} = L_1 + L_2 + \dots L_n \quad (16)$$

### C. Inductors in Parallel

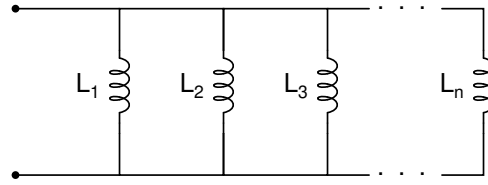


Fig. 11: Inductors in Parallel

$$L_P = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \dots \frac{1}{L_n}} \quad (17)$$

### D. Terminal Equations for Ideal Inductors

Inductance is a linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the field. Inductors are passive elements; they can store and release energy, but they cannot generate or dissipate energy. An inductor:

- does not permit an instantaneous change in its terminal **current**,
- does permit an instantaneous change in its terminal **voltage**, and
- behaves as a short circuit in the presence of a constant terminal voltage.

$$v = L \frac{di}{dt} \quad (\text{V}) \quad (18)$$

$$i = \frac{1}{L} \int_0^t v dt + i(0) \quad (\text{A}) \quad (19)$$

$$p = vi = Lv \frac{di}{dt} \quad (\text{W}) \quad (20)$$

$$w = \frac{1}{2} L i^2 \quad (\text{J}) \quad (21)$$

## VIII. POWER

Power Triangle with power at the top and current and voltage at the bottom. Again, this arrangement represents the actual position of each quantity within the Ohms law power formulas.

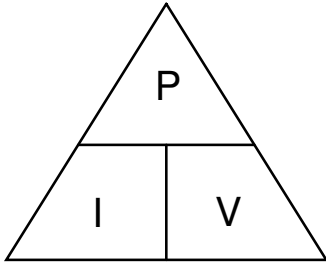


Fig. 12: The Power Triangle

$$P = IV \quad I = \frac{P}{V} \quad V = \frac{P}{I} \quad (22)$$

There are also power equations for if you only want to find power (P). Power is measured in Watts (W).

$$P = I^2 R \quad P = VI \quad R = \frac{V^2}{P} \quad (23)$$

## IX. ADDITIONAL ANALYSIS TECHNIQUES

### A. Nodal Analysis

Take the voltage from the back of the arrow minus the voltage from the front of the arrow.

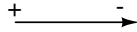


Fig. 13: Flow of current arrow

## X. AC CIRCUIT ANALYSIS

### A. Degrees & Radians

$$(\text{degrees}) = \frac{180^\circ}{\pi} (\text{radians}) \quad (24)$$

$$(\text{radians}) = \frac{\pi}{180^\circ} (\text{degrees})$$

### B. Sine & Cosine

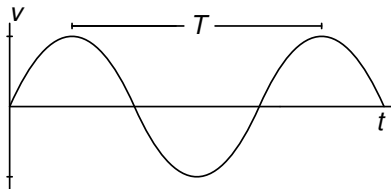


Fig. 14: A sinusoidal voltage

$$\omega = \frac{2\pi}{T} = 2\pi f \quad f = \frac{1}{T} \quad (25)$$

We can express a sinusoidally varying function with either the sine function or the cosine function.

$$v = V_m \cos(\omega t + \phi) \quad (26)$$

### C. Calculating $V_{rms}$

An important characteristic of the sinusoidal voltage (or current) is its rms values. The rms value of a periodic function is defined as the square root of the mean value of the squared function. Hence, if  $v = V_m \cos(\omega t + \phi)$ , the rms value of  $v$  is

$$V_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) dt} \quad (27)$$

The rms value of a sinusoidal voltage depends only on the maximum amplitude. The rms value is not a function of either the frequency or the phase angle.

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad (28)$$

### D. Calculating Phase delay in degrees

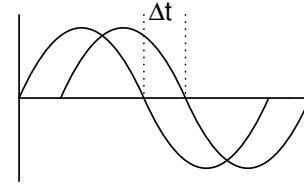


Fig. 15: Phase Shift

$$\Delta\theta^\circ = \Delta t \times \frac{360^\circ}{T} \quad (29)$$

$\Delta\theta^\circ = \text{phase shift in degrees}$

### E. Euler Formula

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta \quad (30)$$

### F. Rectangular and Polar Coordinates

A complex number  $z = x + jy$  is the ordered pair  $(x, y)$  of real numbers, which can be considered as the Cartesian coordinates of a point in the  $xy$ -plane.

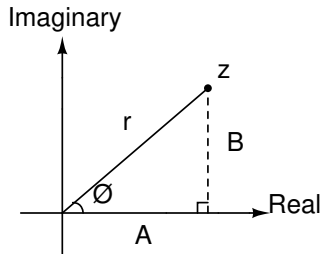


Fig. 16: Complex Plane

The following is the equation in rectangular coordinates:

$$Z = A + Bj \quad (31)$$

The following is the equation in polar coordinates:

$$Z = r \angle \theta^\circ \quad (32)$$

To convert from rectangular coordinates to polar coordinates:

$$r = \sqrt{A^2 + B^2} \quad \theta = \tan^{-1} \left( \frac{B}{A} \right) \quad (33)$$

To convert from polar coordinates to rectangular coordinates:

$$A = r \cdot \cos(\theta) \quad B = r \cdot \sin(\theta) \quad (34)$$

To type polar coordinates into your calculator follow this guide:

Set your calculator to the following MODE:

- DEGREE
- POLAR
- a+bi

Type the following into your calculator to calculate in polar form:

$$5 \angle -53^\circ V = 5e^{-53\pi i / 180} \quad (35)$$

Use the following to help identify time domain and frequency domain:

TIME DOMAIN	FREQUENCY DOMAIN
$A \cos(\omega t \pm \theta)$	$A \angle \pm \theta^\circ$
$A \sin(\omega t \pm \theta)$	$A \angle \pm \theta^\circ - 90^\circ$

The following are examples of converting from voltage functions to phasors.

$$\begin{aligned} v_1(t) &= 12 \cos(377t - 425^\circ) V \longrightarrow V_1 = 12 \angle -425^\circ V \\ v_2(t) &= 18 \sin(2513t + 4.2^\circ) V \longrightarrow V_2 = 18 \angle -85.8^\circ V \end{aligned} \quad (36)$$

The following are conversions from phasors to the time domain if the frequency was 400 Hz.

$$\begin{aligned} V_1 &= 10 \angle 20^\circ V \longrightarrow v_1(t) = 10 \cos(800\pi t + 20^\circ) V \\ V_2 &= 12 \angle -60^\circ V \longrightarrow v_2(t) = 12 \cos(800\pi t - 60^\circ) V \end{aligned} \quad (37)$$

### G. Impedance, Admittance, Reactants

Component	Impedance	Reactance
R	$\vec{Z}_R = R$	$X_R = 0$
L	$\vec{Z}_L = j\omega L$	$X_L = \omega L$
C	$\vec{Z}_C = \frac{1}{j\omega C}$	$X_C = \frac{1}{\omega C}$

## XI. RESPONSE OF FIRST-ORDER RL AND RC CIRCUITS

### A. The Natural Response of an RL Circuit

The current in inductor(L) cannot change instantaneously. Voltage in inductor can.

$$\tau = \text{time constant} = \frac{L}{R} \quad (38)$$

$$i(t) = I_0 e^{-t/\tau}, \quad t \geq 0$$

$$v(t) = I_0 R e^{-t/\tau}, \quad t \geq 0^+ \quad (39)$$

$$p = I_0^2 R e^{-2t/\tau}, \quad t \geq 0^+$$

$$w = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}), \quad t \geq 0$$

### B. The Natural Response of an RC circuit

The current in capacitor (C) can change instantaneously. Voltage cannot.

$$\tau = RC \quad (40)$$

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}, \quad t \geq 0^+$$

$$p = vi = \frac{V_0^2}{R} e^{-2t/\tau}, \quad t \geq 0^+ \quad (41)$$

$$w = \int_0^t p dx = \int_0^t \frac{V_0^2}{R} e^{-2x/\tau} dx$$

$$= \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}), \quad t \geq 0$$

## XII. STEP RESPONSE OF RL AND RC CIRCUIT

### A. Step Response of RL Circuit

$$i(t) = \frac{V_s}{R} + \left( I_0 - \frac{V_s}{R} \right) e^{-(R/L)t} \quad (42)$$

$$v(t) = (V_s - I_0 R) e^{-(R/L)t} \quad (43)$$

### B. Step Response of RC Circuit

$$v_C = I_s R + (V_0 - I_s R) e^{-t/RC} \quad t \geq 0 \quad (44)$$

$$i = \left( I_s - \frac{V_0}{R} \right) e^{-t/RC} \quad t \geq 0^+ \quad (45)$$

#### 1) General Solution:

$$x(t) = x_f + [x(t_0) - x_f] e^{-(t-t_0)/\tau} \quad (46)$$

## XIII. NATURAL AND STEP RESPONSE OF RLC CIRCUIT

$$\alpha = \frac{1}{2RC} \quad (\text{Neper Freq.}) \quad (47)$$

$$\omega_0^2 = \frac{1}{LC} \quad (\text{Resonant Radian Freq.})$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad (\text{Characteristic Roots}) \quad (48)$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Outcomes		
Over-damped	$\omega_0^2 < \alpha^2$	$s_1 \& s_2$ are real distinct
Under-damped	$\omega_0^2 > \alpha^2$	$s_1 \& s_2$ are complex conj.
Critically Damped	$\omega_0^2 = \alpha^2$	$s_1 = s_2$ are real distinct

## XIV. UNDERSTANDING DECIBLES(DB)

Decibel (dB), unit for expressing the ratio between two physical quantities, usually amounts of acoustic or electric power, or for measuring the relative loudness of sounds. One decibel (0.1 bel) equals 10 times the common logarithm of the power ratio.

### A. Power dB

$$N_{dB} = 10 \log_{10} \left( \frac{P_2}{P_1} \right) \quad \begin{array}{l} P_2 = \text{output power} \\ P_1 = \text{input power} \end{array} \quad (49)$$

### B. Voltage dB

$$N_{dB} = 20 \log_{10} \left( \frac{X_2}{X_1} \right) \quad \begin{array}{l} X_2 = \text{output voltage or current} \\ X_1 = \text{input voltage or current} \end{array} \quad (50)$$

### C. Inverse dB

The following equation is going from  $N_{dB}$  to  $\left( \frac{X_2}{X_1} \right)$ .

$$\left( \frac{X_2}{X_1} \right) = 10^{(N_{dB}/20)} \quad (51)$$

## XV. UNDERSTANDING BODE PLOTS

## XVI. DIGITAL CIRCUITS

### A. Digital-to-Analog Conversion

$$v_0 = (b_1 2^{-1} + b_2 2^{-2} + \dots + b_n 2^{-n}) V_{FS} \quad \text{for } b_i \in \{1, 0\} \quad (52)$$

#### 1) Quantization Error:

$$v_\epsilon = |v_X - (b_1 2^{-1} + b_2 2^{-2} + \dots + b_n 2^{-n}) V_{FS}| \quad (53)$$